DESCRIBING RECENTLY DISCOVERED NARROW STATES AS QUARKONIA USING A POTENTIAL MODEL*

STANLEY F. RADFORD

Department of Physics, Marietta College
Marietta, Ohio 45750, USA

WAYNE W. REPKO

Department of Physics and Astronomy, Michigan State University
East Lansing, Michigan, 48824, USA

We examine to what extent several recently discovered narrow resonances can be interpreted as conventional $c\bar{c}$ bound states describable using a potential model. In doing so, we use a semirelativistic approach, which includes both the v^2/c^2 and QCD one-loop corrections to the short distance potential and a long range linear potential together with its scalar and vector v^2/c^2 spin-dependent terms.

1. Introduction

With the recent experimental results for several expected states (η_C' and h_C) in the charmonium spectrum, and the discovery of a state ($X(3872)$), which could be a $^3D_2$ charmonium level, it seems an appropriate time to revisit the potential model interpretation of the $c\bar{c}$ spectrum. For such models, the challenges seem to be:

- Are potential models capable of describing the spin splitting in a quantitatively satisfactory way?
- Including the additional data, is it possible to determine the Lorentz properties of the phenomenological confining potential?
- How well are the leptonic and radiative decays predicted?
- Under what circumstances can the state at 3872 MeV be interpreted as a charmonium level?

Here, we will attempt to answer these questions using a potential model which includes the v^2/c^2 and all one-loop corrections to the short distance potential supplemented with a linear phenomenological confining potential and its v^2/c^2 corrections.

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2. Potential Models

Potential models range from non-relativistic forms such as the Cornell model [1]

$$\mathcal{H}_C = \frac{p^2}{2m} + Ar - \frac{4\alpha_S}{3r}\,, \tag{1}$$

recently used to identify new charmonium states observable in B decays [2], to those including all v^2/c^2 spin-dependent corrections, [3,4] $\mathcal{H} = \mathcal{H}_C + V_{HF} + V_{LS} + V_{TEN}$, which can be used to analyze the spectrum and decays of charmonium [5]. One-loop QCD corrections can then be added to complete the non-relativistic treatment.[6]

In our analysis, we utilize a semi-relativistic Hamiltonian of the form

$$\mathcal{H} = 2\sqrt{p^2 + m^2} + Ar - \frac{4\alpha_S}{3r}F(\mu, r) + V_S + V_L = \mathcal{H}_0 + V_S + V_L\,. \tag{2}$$

Explicit forms of the short distance potential V_S, the long distance potential V_L and $F(\mu, r)$, the one-loop QCD correction to $4\alpha_S/3r$, can be found in Ref. 7. The $c\bar{c}$ mass spectrum and the corresponding wave functions were obtained using a variational approach. The wave functions were expanded as

$$\psi_\ell^m(\vec{r}) = \sum_{n=0}^{N} C_n \left(\frac{r}{R}\right)^n e^{-r/R} Y_\ell^m(\Omega)\,, \tag{3}$$

and the C_n's were determined by minimizing $E = \langle\psi\,|\mathcal{H}\,|\psi\rangle/\langle\psi\,|\psi\rangle$. This procedure results in a linear eigenvalue equation for the C_n's and the energies. The wave functions corresponding to different eigenvalues are orthogonal and the jth eigenvalue λ_j is an upper bound on the exact energy E_j. For $N = 10$, the lowest four eigenvalues are stable to a part in 10^6.

3. Results and Conclusions

The energies and wave functions were obtained by treating $V_S + V_L$ as a perturbation to $\mathcal{H}_0$ and by treating $\mathcal{H}$ nonperturbatively. We fit the spectrum to ten well established [a] $c\bar{c}$ states by adjusting the parameters A, α_S, m, μ and the vector fraction f_V to minimize χ^2. Our nonperturbative fit gives: $A = 0.175\,\text{GeV}^2, \alpha_S = 0.361, m = 1.49\,\text{GeV}, \mu = 1.07\,\text{GeV}$ and $f_V = 0.18$. Interestingly, the perturbative fit yields similar results for A, α_S and m, but prefers $\mu = 2.32\,\text{GeV}$ and $f_V = 0$. The results for the levels are given in Table 1 and the predicted E_1 transitions widths are given in Table 2.

The semi-relativistic model provides a quantitatively good description of the charmonium spectrum. Of the states included in the fit, only the $^3D_1(3770)$ is poorly described. The E_1 widths agree reasonably well with experiment[8]. However, based on the model considered here, the $X(3872)$ cannot be explained solely in terms of a charmonium $^3D_2$ state described by a potential. Spin effects alone can only separate the $^3D_2$ from the $^3D_1$ by 40 MeV or so, which suggests that the inclusion of open channel effects is essential if this identification is to be established.

[a]$1^1S_0, 1^3S_1, 1^3P_J, 2^1S_0, 2^3S_1, 1^3D_1, 3^3S_1$ and 2^3D_1.

Table 1.   Perturbative and nonperturbative results for the $c\bar{c}$ spectrum.

	Pert	Non-pert	Expt		Pert	Non-pert	Expt
η_c	2985	2981	2979.7 ± 1.5	η_c'	3599	3624	(3637.7 ± 4.4)
J/ψ	3096.9	3096.9	3096.87 ± 0.04	ψ'	3686	3686	3686.0 ± 0.1
χ_0	3418.4	3415.8	3415.1 ± 0.8	χ_0'	3849	3872	
χ_1	3510.2	3510.4	3510.51 ± 0.12	χ_1'	3946	3951	
χ_2	3556.5	3556.3	3556.18 ± 0.17	χ_2'	3999	3996	
h_c	3527	3524	(3526.21 ± 0.25)	h_c'	3966	3966	
1^3D_1	3809	3790	3770 ± 2.5	2^3D_1	4174	4157	4160 ± 20
1^3D_2	3827	3826	3872 ± 1.0	2^3D_2	4198	4201	
1^3D_3	3831	3845		2^3D_3	4209	4223	
1^1D_2	3824	3825	3836 ± 13.0	2^1D_2	4199	4202	

Table 2. E_1 transition widths.

$\Gamma_\gamma(E1)$ (keV)	TH	EX	$\Gamma_\gamma(E1)$ (keV)	TH	EX
$\chi_0 \to \gamma J/\psi$	169	119 ± 17	$1^3D_2(3826) \to \gamma\chi_1$	314	
$\chi_1 \to \gamma J/\psi$	357	288 ± 51	$1^3D_2(3826) \to \gamma\chi_2$	76.3	
$\chi_2 \to \gamma J/\psi$	468	426 ± 48	$1^3D_2(3872) \to \gamma\chi_1$	459	
$h_c \to \gamma\eta_c$	670		$1^3D_2(3872) \to \gamma\chi_2$	119	
$\psi' \to \gamma\chi_0$	22	24.2 ± 2.5	$\psi(3770) \to \gamma\chi_0$	291	320 ± 100
$\psi' \to \gamma\chi_1$	33	23.6 ± 2.7	$\psi(3770) \to \gamma\chi_1$	125	280 ± 100
$\psi' \to \gamma\chi_2$	29	24.2 ± 2.5	$\psi(3770) \to \gamma\chi_2$	5.6	≤ 330
$\eta_c' \to \gamma h_c$	22				

Acknowledgments

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